MATH 147 QUIZ 2 SOLUTIONS

1. For the function $f(x, y)$, the point $(a, b) \in \mathbb{R}^2$, define the partial derivative of $f(x, y)$ with respect to y at (a, b). (2 Points)

   We define the partial derivative of $f(x, y)$ w.r.t y as

$$\frac{\partial f}{\partial y}(a, b) = \lim_{h \to 0} \frac{f(a, b+h) - f(a, b)}{h}.$$

2. For $f(x, y) = \begin{cases} \frac{3x^2y - y^3}{x^2+y^2}, \text{ if } f(x, y) \neq (0, 0) \\ 0, \text{ if } (x, y) = (0, 0), \end{cases}$ find a formula for $f_x(x, y)$.(4 points)

   As this is a rational function in x, it is differentiable everywhere except possibly for when $x^2 + y^2 = 0$. Thus, we first see what the function is doing away from the origin. Use the quotient rule to see

$$f_x(x, y) = \frac{(x^2 + y^2)(6xy) - (3x^2y - y^3)(2x)}{(x^2 + y^2)^2} = \frac{8xy^3}{(x^2 + y^2)^2}.$$

   Next, we use the limit definition of derivative to see what is happening at the origin. We should have

$$f_x(0, 0) = \lim_{h \to 0} \frac{f(h, 0) - f(0, 0)}{h} = \lim_{h \to 0} \frac{\frac{0}{h^2} - 0}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = 0.$$

   Thus, we can say that

$$f_x(x, y) = \begin{cases} \frac{8xy^3}{(x^2+y^2)^2}, \text{ if } (x, y) \neq (0, 0) \\ 0, \text{ if } (x, y) = (0, 0). \end{cases}$$

3. Find the tangent plane to the graph of $z = f(x, y) = -9x^3 - 3y^2$ at the point $(2, 1, f(2, 1))$. (4 points)

   We know that the equation for a tangent plane is $z = f_x(a, b)(x - a) + f_y(a, b)(y - b) + f(a, b)$. Thus we first find the partial derivatives: $f_x(x, y) = -27x^2$ and $f_y(x, y) = -6y$, so we have $f_x(2, 1) = -108$ and $f_y(2, 1) = -6$. We also see that $f(2, 1) = -75$. Putting this together, we have that the equation for the tangent plane to the function $f(x, y)$ at $(2, 1)$ is given by the equation

$$z = -108(x - 2) - 6(y - 1) - 75.$$